Probability and Computing
Spring 2019
HW8: Redo Midterm 2

This assignment is due **Friday, May 3 in class**. Please submit **both** your original midterm and your corrected solutions.

1 Finding the Maximum

Let X and Y be random variables, where $X \sim \text{Unif}(0,1)$, $Y \sim \text{Unif}(0,1)$, and $X \perp Y$. Let $Z = \max\{X, Y\}$.

(a) [6 pts] What is $\mathbf{Pr}\{Z < t\}$, for $0 < t < 1$?

(b) [6 pts] What is $f_Z(t)$? (Use your answer from part (a) to find this. If you don't have a solution to part (a), you can use $\mathbf{Pr}\{Z < t\} = t^4$.)

(c) [8 pts] What is $\mathbf{E}[Z]$? (Use your answer from part (b) to find this. If you don't have a solution to part (b), you can use $f_Z(t) = \frac{4}{3} - t^2$.)

(d) [10 pts] What is $\mathbf{E}[Z \mid Y = 1/2]$?
Hint: condition on whether $X < 1/2$ or $X > 1/2$.

2 SITA Dispatching

Imagine you have a queueing system with three servers, each of which has its own queue. Interarrival times are Exponentially distributed with rate $\lambda = 1/2000$, and service times are Exponentially distributed with rate $\mu = 1/1000$. When a job arrives to the system, we send it to one of the three servers, each with equal probability.

(a) [8 pts] What is the mean response time in this system?

(b) [10 pts] Now suppose that when a job arrives to the system, instead of randomly selecting where to send it we use the "Size Interval Task Assignment" (SITA) policy, which does the following. "Small" jobs, defined to be jobs with size $S < 500$, are sent to server 1. "Medium" jobs, defined to be jobs with size $500 < S < 1500$, are sent to server 2. "Large" jobs, defined to be jobs with size $S > 1500$, are sent to server 3.

What is the expected size of a job that has been sent to server 2? (Set up an expression, but don't solve/simplify it.)

Hint: We did an example almost identical to this in class.

3 Multi-Task Jobs

Suppose that my system has two types of jobs: type X and type Y. Type Y jobs consist of a single task, which takes time Y_1 to run. Type X jobs consist of n tasks, which must be completed sequentially, and where the first task takes time X_1, the second task takes time X_2, and so on. A type X job is only complete once all n of its tasks have completed.

Assume that Y_1 is Exponentially distributed with rate a and all of the X_i's are Exponentially distributed with rate b.

Hint: Assuming you've memorized the expected value of an exponential, you should NOT need to do any integrals on this problem!

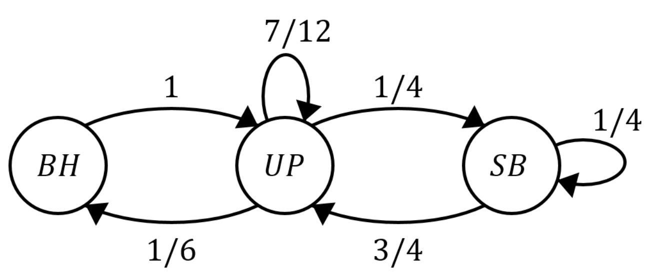
(a) [8 pts] What is the expected time required for a type X job?

(b) [12 pts] Suppose that a type Y job and a type X job start running at exactly the same time. What is the probability that the type X job finishes first?

4 Backhoes and Bugs

Data centers are sometimes "up," or working, and they are sometimes "down." There are many reasons why a data center could be down, two of which are (1) a software bug caused the system to fail (SB), and (2) a backhoe dug up some cable (BH). Suppose a data center that is working today will be down tomorrow due to backhoe reasons with probability $\frac{1}{6}$, and will be down tomorrow due to a software bug with probability $\frac{1}{4}$. A data center that is down today for backhoe reasons will be back up tomorrow with probability 1. A data center that is down today due to a software bug will be back up tomorrow with probability $\frac{3}{4}$, and otherwise still will be experiencing a software failure.

Here's a Markov chain describing this system:

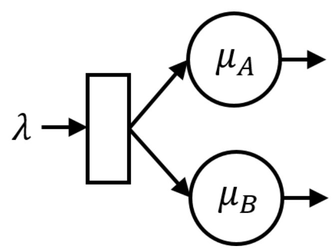


(a) [15 pts] Solve this Markov chain to find the stationary distribution of the system state.

(b) [5 pts] Suppose the company loses \$100 for every day that the data center is down. What is the expected monthly loss? (You can assume a month has exactly 30 days.)

5 M/M/2/3

An M/M/2/3 system is similar to an M/M/2 (so interarrival times and service times are Exponetially distributed, and there are two servers), with one restriction: the system only has space for 3 jobs at a time. That is, there can be at most one job waiting in the queue. If a new job arrives and the system is full, the new job will be rejected and can't enter the system. Assume that server A has rate μ_A and server B has rate μ_B. Assume that when a job arrives to an empty system, it flips a coin and goes to server A with probability p. Here's what our system looks like:



[15 pts] Draw a Markov chain for this system. (Don't solve the Markov chain.)

Hint: Make sure your states keep track of all the information you might need about the system.