The purpose of this assignment is to help me understand the range of math backgrounds of students in this course. This is a required assignment, but it will not be graded for correctness; you will receive full credit for turning in an honest attempt to solve the problems. Please show your work. I do not expect you to know how to solve all of these problems. If you have never seen something before and have no idea how to approach it, feel free to write “I don’t know.”

Do not look on the internet (or any other source) for direct answers to these problems, but you may use the internet, textbooks, notes from previous classes, etc. to refresh your memory on material that you have learned before but don’t remember perfectly.

1 Problems

1. What is \( (n^n) \)? Write an expression for \( (n^n) \).

2. Derive a closed-form expression for \( S \) where \( S = 1 + x + x^2 + x^3 + \cdots + x^n \).

3. Derive a closed-form expression for \( S = \sum_{i=0}^{\infty} x^i \), where \( |x| < 1 \).

4. Derive a closed-form expression for \( S = 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} \).

5. Derive a closed-form expression for \( S = \sum_{i=1}^{\infty} i x^{i-1} \), where \( |x| < 1 \).

6. Derive a closed form expression for \( \sum_{i=1}^{n} (n^n) x^i y^{n-i} \).

7. What is \( \lim_{n \to \infty} (1 + \frac{1}{n})^n \)?

8. What is \( \lim_{n \to \infty} (1 + \frac{2}{n})^n \) (assume \( x \) is a constant)?

9. Write \( e^x \) using the Taylor series expansion.

10. Evaluate: \( \int_{t=a}^{x} \frac{1}{a} dt \)

11. Evaluate: \( \int_{y=0}^{\infty} ye^{-y} dy \)

12. Evaluate: \( \int_{y=0}^{\infty} \int_{x=0}^{y} e^{-y} dx \ dy \). Do this twice, once without changing the order of integration and once changing the order of integration.

13. Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \). Compute \( A^2 \).