COSC-450: Notes for HW5

4.3 asks you to simulate an M/M/1 queue. You should check the mean response times produced by your simulator against the formula $\frac{1}{\mu-\lambda}$ that we computed in class for the M/M/1. This is mostly for practice, because your simulator will be set up in the same way as your simulator for 20.1. Doing this first for the M/M/1 will help you check that your simulator is set up properly, given that we have an easy analytical formula that should give you the same result as your simulator.

20.1 asks you to do more or less the same thing, but in a M/BP/1 queue—that is, service times have a Bounded Pareto distribution—and for time in queue rather than response time. We didn't quite get to the Bounded Pareto in class today. It is very similar to the Pareto distribution, but with a lower bound, k, and an upper bound, p. The formal definition is in Section 20.4 of the book. Part (b) of 20.1 asks you to compute the analytical mean and variance of time in queue. For this you'll need the first three moments of the BP distribution; you'll probably want to use Mathematica or Wolfram Alpha or some other math software to compute these more easily.

For both of these problems, you will need to write code to generate random variables from different distributions. Chapter 4 has some techniques for how to do this. One of these, which we'll use here, is called the "inverse transform method." The idea is to generate an instance of a Uniform(0,1), which is easy to do on a computer, and then plug that in to the inverted cdf.

For an exponential, do the following:
1. Generate $u \in U(0,1)$
2. Set $x = -\frac{1}{\lambda}\ln(1-u)$. This gives x, an instance of $X \sim \text{Exp}(\lambda)$.

For a Bounded Pareto, $\text{BP}(k,p,\alpha)$, do the following:
1. Generate $u \in U(0,1)$
2. Set

$$x = \frac{k}{\left(1+u\left(\left(\frac{k}{p}\right)^{\alpha}-1\right)\right)^{1/\alpha}}.$$

This gives x, an instance of $X \sim \text{BP}(k,p,\alpha)$.