COSC-450: Analytical Performance Modeling
Probability Review

Discrete distributions are defined by a *probability mass function*, $p_X(a)$. How is this function defined?

What is $\sum_a p_X(a)$?

Continuous distributions are defined by a *probability density function*, $f_X(x)$. How is this function defined?

What is $\int_{-\infty}^{\infty} f_X(x)$?

What is $\int_a^b f_X(x)$?

Here are some common discrete distributions, each of which can be described as the result of some coin-flipping experiment. Fill out the table below:

Distribution	**Coin flipping experiment**	**pmf**
Bernoulli(p)		
Binomial(n, p)		
Geometric(p)		

Here are some common discrete distributions. Fill out the table below:

Distribution	**Coin flipping experiment**	**pmf**
Uniform(a, b)		
Exponential(λ)		

How do you compute the expectation and variance of a random variable X...

	Expectation	**Variance**
...if X is discrete?		
...if X is continuous?		

1 Computing Expectation and Variance via Conditioning 1

A student, in search of a job, tries two approaches: diligent and lazy. When the student tries the diligent approach, she ends up spending 19 hours preparing for her interview, which will result in a job offer with probability 0.05. When she tries the lazy approach, she spends one hour preparing for her interview, and she never gets a job offer. So far, the student has not received a job offer, so she cannot tell which approach works better. She therefore decides before each interview to choose an approach (diligent or lazy) each with probability 1/2.

(a) Assuming the student starts searching for a job today, what is the expected number of hours she'll spend preparing for interviews?

(b) Compute the variance on the amount of time the student ends up spending preparing for interviews.

2 Computing Expectation and Variance via Conditioning 2

My friend Stefan enjoys commuting home from work by taking a random walk around the city of Pittsburgh (true story). When he leaves his office, he walks along a road until he reaches an intersection, then flips two coins to decide whether to turn right, turn left, continue in his current direction, or turn around (Pittsburgh has many non-four-way intersections, so I actually think Stefan uses a random number generator app to select his path. And he allows himself to turn around automatically if he's heading towards an unsafe neighborhood.) He continues this until he finds himself at home.

Let's simplify Stefan's situation a little. Suppose that when he leaves his office, he goes right or left with equal probability. If he goes right, with probability $\frac{4}{5}$ he'll be back at his office after wandering around for 10 minutes, and with probability $\frac{1}{5}$ he'll be home after walking for 15 minutes. If he goes left, with probability $\frac{2}{3}$ he'll wander for 6 minutes before returning to his office, and with probability $\frac{1}{3}$ he'll wander for 21 minutes before returning to his office.

Let T denote the time it takes for Stefan to get home. What's $E[T]$, and what's $\mathrm{Var}(T)$?

3 Linearity of Expectation

When I was in high school, one of my friends hosted a hat party that n people attended. The premise was as follows: each person was to bring a unique-looking hat to the party. All of the hats would be thrown in a bag, and then each person would pick a random hat out of the bag and wear it for the rest of the evening. What's the expected number of people who end up wearing the hat that they brought to the party?