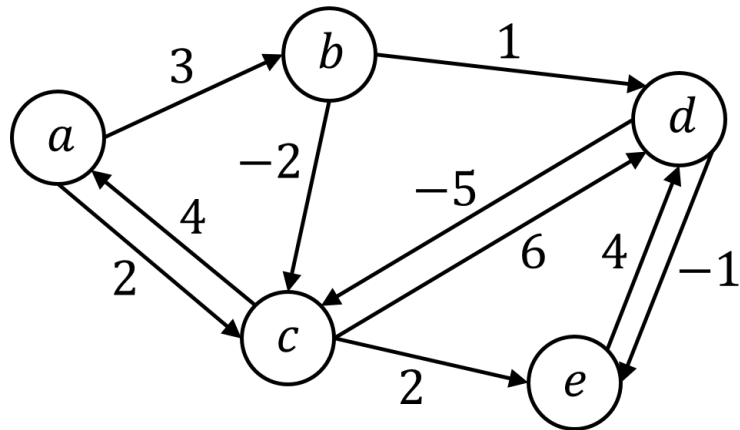


# COSC 311: ALGORITHMS

## MINI 8

Due Friday, November 8 in class

1. Shortest paths revisited. Here's a graph:



Fill in the below table to show what happens when you run the Bellman-Ford algorithm on this graph to find the shortest path from  $a$  to each other node.

**Solution:**

	0	1	2	3	4
$a$	0	0	0	0	0
$b$	$\infty$	3	3	3	3
$c$	$\infty$	2	1	-1	-1
$d$	$\infty$	$\infty$	4	4	4
$e$	$\infty$	$\infty$	4	3	1

**2. Reconstructing solutions.** Explain how to use the filled-in table from problem 1 to reconstruct the sequence of nodes visited on the shortest path from  $a$  to  $e$ .

**Solution:** We begin in row  $e$ , column 4, which stores the cost of the shortest path from  $a$  to  $e$ . This value is 1, which must be equal to either (1) the shortest path from  $a$  to  $e$  using at most 3 edges (stored in row  $e$ , column 3 of the table), or (2) the cost of some edge  $(x, e)$  into  $e$ , plus the shortest path from  $a$  to  $x$ . There are two options here:  $(c, e)$ , which has cost 2, plus the entry in row  $c$  and column 3, which is -1; and  $(d, e)$ , which has cost -1, plus the entry in row  $d$  and column 3, which is 4. The  $(c, e)$  option matches the value stored in row  $e$ , column 4, so we know that edge  $(c, e)$  is part of the shortest path to  $e$ .

We continue tracing backwards from row  $c$ , column 3. We have four options: (1) row  $c$ , column 2: cost 1; (2) edge  $(a, c)$  plus row  $a$ , column 2: cost 2; (3) edge  $(b, c)$  plus row  $b$ , column 2: cost 1; (4) edge  $(d, c)$  plus row  $d$ , column 2: cost -1. The  $(d, c)$  option matches the value stored in row  $c$ , column 3, so we add  $(d, c)$  to our solution path.

We continue backwards in this manner and find that edges  $(b, d)$  and  $(a, b)$  are also part of the shortest path from  $a$  to  $e$ .