COSC 311: ALGORITHMS
MINI 3

Due Monday, September 24 in class

1. Quicksort. Here's an unsorted array. Run quicksort on the array, showing what the array looks like *after* each partition step is complete, assuming you are using the last element as the partition element (you should *not* show the intermediate steps taken while partitioning). You can show the left and right recursions in the same picture.

2	11	6	14	1	10	4	7	16	15	12	3	8	5	13	9

Solution:
Bold and underlined numbers indicate those positions which are fixed on each level of the recursion.

2	6	1	4	7	3	8	5	**9**	15	12	10	14	11	13	16
2	1	4	3	**5**	6	8	7	**9**	15	12	10	14	11	13	**16**
2	1	**3**	4	**5**	6	**7**	8	**9**	12	10	11	**13**	15	14	**16**
1	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	10	**11**	12	**13**	**14**	15	**16**

2. Induction. Suppose you were going to prove by induction that a certain claim holds for all powers of 2, starting with 1. What steps would you take to prove this? Explain in your own words why those steps form a complete, convincing proof.

Solution:

a. *Base Case:* I will show the claim holds for $n_0 = 1$.
b. *Induction Step:* Assuming the claim holds for $n = 2^k$ (*Induction Hypothesis*), I'll show that the claim then holds for $n = 2^{k+1}$.

Explanation: Having proven base case, I showed the claim holds for $n = 2^0$.
Then I know from the proof of my inductive case that the claim will also hold for $n = 2^{(0+1)} = 2^1$.
Applying the inductive step on $n = 2^1$ again, I know the claim holds for $n = 2^{1+1} = 2^2$.
and so on ......
The same logic can be applied to show that the claim holds for any positive integer powers of 2.
Note that I do not have to show directly that the claim holds for 2, 4, 8, and so on. The inductive step takes care of all of these cases at once!