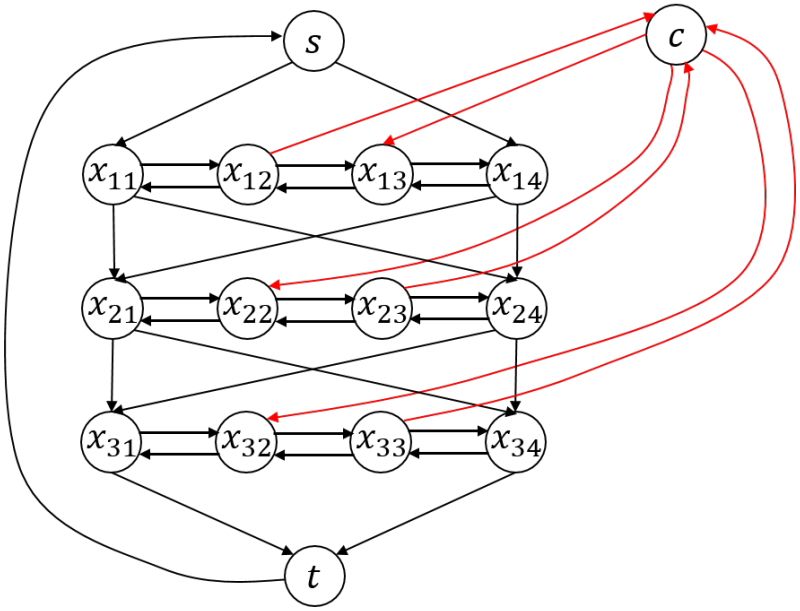
COSC 311: ALGORITHMS
MINI 12

Due Wednesday, December 12 in class

Here's a (very tiny) instance of 3-SAT:

$$x_1 \vee \bar{x_2} \vee \bar{x_3}$$

And here's a graph corresponding to the Hamiltonian Cycle instance we created in our reduction (some of the edges are in red just to make the graph easier to read):



This mini homework is about showing that there's a mapping between "yes" instances of 3-SAT and "yes" instances of Hamiltonian cycle.

1. Find a Hamiltonian cycle in this graph. Explain how you can use this cycle to find a satisfying assignment for the original 3-SAT instance.

(There can be other valid Hamiltonian cycles)
$s - x_{11} - x_{12} - c - x_{13} - x_{14}$
$-x_{21} - x_{22} - x_{23} - x_{24}$
$-x_{31} - x_{32} - x_{33} - x_{34} - t - s$

In the Hamiltonian Cycle, if I go through the row for x_i from left to right, then let the value for x_i be `true`, else be `false`.
So, for the given Hamiltonian Cycle above, the value of variables are: $x_1 =$ `true`, $x_2 =$ `true`, $x_3 =$ `true`.

2. One possible satisfying assignment for the 3-SAT instance is $x_1 = \texttt{false}$, $x_2 = \texttt{true}$, $x_3 = \texttt{false}$. Explain how you can use this satisfying assignment to find a Hamiltonian cycle in the graph created in the reduction.

1. Always start from s and progress row by row
2. for the i^{th} row, go from left to right if x_i is `true`; go from right to left if x_i is `false`.
3. *Whenever possible*, take the detour to visit c_j and come back to next node the same row.
4. After finishing x_n (the last variable row), visit t then go back to s.

Whenever possible: take the detour to visit c_j iff:
a. c_j has not been visited.
b. the current direction make the visit to c_j possible.

So for the given assignment the Hamiltonian Cycle would be
$s - x_{14} - x_{13} - x_{12} - x_{11}$
$-x_{21} - x_{22} - x_{23} - x_{24}$
$-x_{34} - x_{33} - c - x_{32} - x_{31} - t - s$