Name: ____

COSC-311: Algorithms
Fall 2017
Final Exam

This is a 50-minute, closed-notes exam. There are four problems, each of which is worth 25 points. Good luck!

Problem	**Score**
1	/25
2	/25
3	/25
4	/25
TOTAL:	**/100**

1 True/False/Unknown

State whether each of the following claims is true, false, or not known (meaning that given the current state of theoretical CS the answer is not known, not that you personally do not know). Give a short explanation of your answer or, if appropriate, a counterexample.

(a) Claim 1: Multiple Interval Scheduling $\leq_P$ Weighted Interval Scheduling.

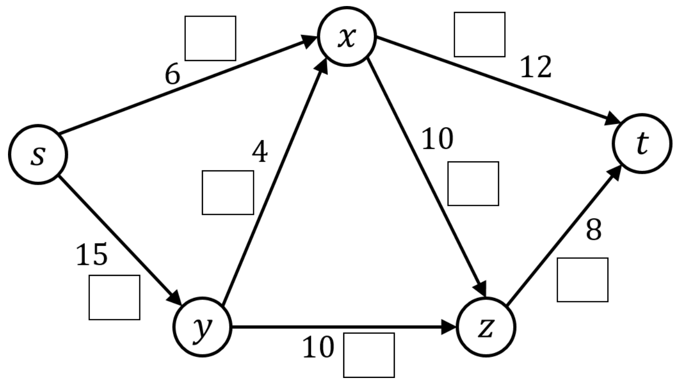
(b) Claim 2: If $X \leq_P Y$ and $Y \in \Omega(2^n)$, then $X \in \Omega(2^n)$.

(c) Let $G = (V, E)$ be a flow network with source s, target t, and positive integer capacities $c(e)$ for each edge $e \in E$. Let (A, B) be a min cut in the graph.

Claim 3: If we add 1 to the capacity of every edge in G, then (A, B) is still a min cut.

2 Max Flows

Here is a flow network; edges in the graph are labeled with their capacities



(a) Find a max flow in the graph. Write the value of the flow you are sending along each edge in the empty box next to the edge.

(b) Draw the residual graph for the max flow you found in part (a).

(c) Explain how to use your residual graph from part (b) to find a min cut in the original graph (and specify which vertices are on each side of the cut that you find in this manner).

3 Latin Squares

A *Latin Square* is an $n \times n$ grid filled with n symbols, where each symbol appears exactly once in each row and in each column. For example, the square:

W	X	Y
X	Y	W
Y	W	X

is a 3×3 Latin Square. As another example, any Sudoku board is a 9×9 Latin Square.

Consider the following problems:

Input:

- A list of n symbols $x_1, \ldots, x_n$
- An $n \times n$ grid that is partially filled using the symbols $x_1, \ldots, x_n$.

Output: Is it possible to fill in the remaining grid entries to form a valid Latin Square?

(a) Explain why the Latin Squares problem defined above is in NP. It should be clear from your explanation that you understand the definition of the class NP.

(b) How would you go about proving that Latin Squares is NP-complete? You should *not* actually provide a proof, just explain what the proof would involve. It should be clear from your explanation that you understand the definition of NP-completeness.

4 Independent Set

Consider the following instance of 3-SAT:

$$(x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_2 \vee x_3 \vee \bar{x}_4) \wedge (x_1 \vee \bar{x}_2 \vee x_4)$$

(a) Use the reduction we discussed in class to turn this instance of 3-SAT into an instance of Independent Set.

(b) Give a solution to the Independent Set instance you created in part (a).

(c) Explain how to translate your solution to Independent Set from part (b) into a solution to the original instance of 3-SAT.