Practice with Induction

1. Prove that for all \( n > 0 \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).

2. Prove that for all \( n > 0 \), \( \sum_{i=0}^{n} x^i = \frac{1-x^{n+1}}{1-x} \) if \( |x| < 1 \). This is called the finite geometric series.

Note that if we take the limit of this series as \( n \to \infty \), we get \( \sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \). This is called the infinite geometric series, and is another useful summation identity.
3. Let \( T(n) = \begin{cases} T(n/2) + c_1 n & n > 1 \\ c_2 & n = 1 \end{cases} \). Prove that \( T(n) \in O(n) \).

4. Let \( T(n) = \begin{cases} 2T(n-1) + n & n > 1 \\ 1 & n = 1 \end{cases} \). Prove that \( T(n) = 2^{n+1} - n - 2 \).