Practice with Asymptotic Notation

****Turn in at the start of class on Monday. This will not be graded, and is simply for your benefit and to let me know whether the class would benefit from further review.****

Def 1. $O(g(n)) = \{ f(n) : \exists c, n_0 > 0 \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \ \forall n \geq n_0 \}$

Def 2. $\Omega(g(n)) = \{ f(n) : \exists c, n_0 > 0 \text{ such that } 0 \leq c \cdot g(n) \leq f(n) \ \forall n \geq n_0 \}$

Def 3. $\Theta(g(n)) = \{ f(n) : \exists c_1, c_2, n_0 > 0 \text{ such that } 0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \ \forall n \geq n_0 \}$

Def 4. $o(g(n)) = \{ f(n) : \forall c > 0, \exists n_0 > 0 \text{ such that } 0 \leq f(n) < c \cdot g(n) \ \forall n \geq n_0 \}$

Def 5. $\omega(g(n)) = \{ f(n) : \forall c > 0, \exists n_0 > 0 \text{ such that } 0 \leq c \cdot g(n) < f(n) \ \forall n \geq n_0 \}$

Recall that there is an easy test that can be used to compare two functions $f(n)$ and $g(n)$: Compute

$$C = \lim_{n \to \infty} \frac{f(n)}{g(n)}.$$

1. Fill in the table:

<table>
<thead>
<tr>
<th>If...</th>
<th>Then...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 0$</td>
<td></td>
</tr>
<tr>
<td>$0 &lt; C &lt; \infty$</td>
<td></td>
</tr>
<tr>
<td>$C = \infty$</td>
<td></td>
</tr>
</tbody>
</table>

2. Prove that $f(n) = 3n + 16 \in \Theta(n)$. 


3. Prove that $\Theta(\log_a n) = \Theta(\log_b n)$.

4. Prove that $f(n) = 2n^2 + c_1 \notin O(n)$. 