

Name: _____

Practice with Asymptotic Notation

Turn in at the start of class on Monday. This will *not* be graded, and is simply for your benefit and to let me know whether the class would benefit from further review.

Def 1. $O(g(n)) = \{f(n) : \exists c, n_0 > 0 \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \forall n \geq n_0\}$

Def 2. $\Omega(g(n)) = \{f(n) : \exists c, n_0 > 0 \text{ such that } 0 \leq c \cdot g(n) \leq f(n) \forall n \geq n_0\}$

Def 3. $\Theta(g(n)) = \{f(n) : \exists c_1, c_2, n_0 > 0 \text{ such that } 0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \forall n \geq n_0\}$

Def 4. $o(g(n)) = \{f(n) : \forall c > 0, \exists n_0 > 0 \text{ such that } 0 \leq f(n) < c \cdot g(n) \forall n \geq n_0\}$

Def 5. $\omega(g(n)) = \{f(n) : \forall c > 0, \exists n_0 > 0 \text{ such that } 0 \leq c \cdot g(n) < f(n) \forall n \geq n_0\}$

Recall that there is an easy test that can be used to compare two functions $f(n)$ and $g(n)$: Compute

$$C = \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}.$$

1. Fill in the table:

If...	Then...
$C = 0$	
$0 < C < \infty$	
$C = \infty$	

2. Prove that $f(n) = 3n + 16 \in \Theta(n)$.

3. Prove that $\Theta(\log_a n) = \Theta(\log_b n)$.

4. Prove that $f(n) = 2n^2 + c_1 \notin O(n)$.