# COSC 311: ALGORITHMS
# MINI 3
## Solutions

**1. Quicksort.** Here's an unsorted array. Run quicksort on the array, showing what the array looks like *after* each partition step is complete, assuming you are using the last element as the partition element (you should *not* show the intermediate steps taken while partitioning). You can show the left and right recursions in the same picture.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 11 | 6 | 15 | 2 | 10 | 4 | 7 | 16 | 13 | 12 | 5 | 8 | 3 | 14 | 1 |

**Solution:**
Bold and underlined numbers indicate those positions which are fixed on each level of the recursion.

First partition around 9:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 6 | 3 | 2 | 8 | 4 | 7 | **<u>9</u>** | 13 | 12 | 16 | 10 | 15 | 14 | 11 |

Now partition around 5 (on the left) and 13 (on the right):

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 4 | 3 | **<u>5</u>** | 8 | 6 | 7 | **<u>9</u>** | 10 | 12 | 11 | **<u>13</u>** | 15 | 14 | 16 |

Now partition around 2 (left-most section), 8 (second section), 10 (third section), and 15 (right-most section):

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **<u>2</u>** | 4 | 3 | **<u>5</u>** | 7 | 6 | **<u>8</u>** | **<u>9</u>** | **<u>10</u>** | 12 | 11 | **<u>13</u>** | 14 | **<u>15</u>** | 16 |

Now all of the "singletons" are at the base case, and we do one more partition in the sections with two numbers—partition around 4, 7, and 12:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **<u>1</u>** | **<u>2</u>** | 3 | **<u>4</u>** | **<u>5</u>** | 6 | **<u>7</u>** | **<u>8</u>** | **<u>9</u>** | **<u>10</u>** | 11 | **<u>12</u>** | **<u>13</u>** | **<u>14</u>** | **<u>15</u>** | **<u>16</u>** |

And now all remaining sections are at the base case, and we're done.

**2. Induction.** Suppose you were going to prove by induction that a certain claim holds for all powers of 2, starting with 1. What steps would you take to prove this? Explain in your own words why those steps form a complete, convincing proof.

**Solution:**

a. *Base Case:* I will show the claim holds for $n_0 = 1$.
b. *Induction Step:* Assuming the claim holds for $n = 2^k$ (*Induction Hypothesis*), I'll show that the claim then holds for $n = 2^{k+1}$.

Explanation: Having proven base case, I showed the claim holds for $n = 2^0$.
Then I know from the proof of my inductive case that the claim will also hold for $n = 2^{(0+1)} = 2^1$.
Applying the inductive step on $n = 2^1$ again, I know the claim holds for $n = 2^{1+1} = 2^2$.
and so on ......
The same logic can be applied to show that the claim holds for any positive integer powers of 2.
Note that I do not have to show directly that the claim holds for 2, 4, 8, and so on. The inductive step takes care of all of these cases at once!