COSC 311: ALGORITHMS
MINI 2 SOLUTIONS
Due Wednesday, September 19 in class

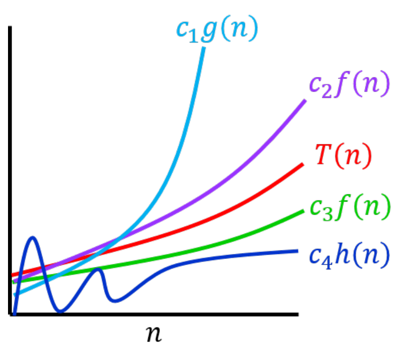
1. Asymptotic comparisons. Order the following functions from slowest-growing to fastest-growing. Some of them may be equal (i.e., may belong to the same Θ class).

$$2n^2 \qquad 65n^{4321} \qquad \lg n \qquad 1700n \qquad 1.5^n \qquad n^3 + 4n^2 + 9000 \qquad n \qquad \frac{1}{1000}n^3$$

Solution:

1. $\lg n$
2. n and $1700n$
3. $2n^2$
4. $\frac{1}{1000}n^3$ and $n^3 + 4n^2 + 9000$
5. $65n^{4321}$
6. 1.5^n

2. Analyzing algorithms. Suppose I wrote an expression, $T(n)$, for the runtime of my algorithm, and then I compared $T(n)$ to some other functions as follows:



Which of the following claims are true about the worst-case performance of my algorithm? (More than one claim may be true).

I. It's $\Theta(f(n))$.

II. It's $\Omega(h(n))$.

III. It's $O(g(n))$, provided $T(n)$ describes the runtime of the worst possible input.

IV. It's $O(g(n))$.

V. It's $\Theta(f(n))$, provided $T(n)$ describes the runtime of the worst possible input.

VI. It's $\Omega(f(n))$.

Solution: II, III, V, and VI are all true.

Some more detail: unless we're told that $T(n)$ describes the runtime of the worst possible input, we don't know what input it is talking about. It could be the runtime for the worst-case input, but it also could be the runtime for the best-case input, or some other input in between.

In case I, we can see from looking at the graph that $T(n) \in \Theta(f(n))$, but there could be some other, worse input that produces a curve that grows much faster. So we can't conclude that the worst-case runtime is $\Theta(f(n))$.

In case II, we can see from the graph that $T(n) \in \Omega(h(n))$. $T(n)$ might not describe the worst-case input, but the worst-case input certainly is at least as bad as $T(n)$, so the worst case must also be $\Omega(h(n))$.

In case III, we are told that $T(n)$ is the worst case, and we can see from the graph that $T(n) \in O(g(n))$.

In case IV, as in case I, we don't know whether $T(n)$ is the worst-case input, so there could be some other worse input that grows faster than $g(n)$.

In case V, we are told that $T(n)$ is the worst case, and since $T(n)$ falls between $c_2 f(n)$ and $c_2 f(n)$ we know that $T(n) \in \Theta(f(n))$.

In case VI, as in case II, we know that $T(n)$ is lower bounded by $c_3 f(n)$ so $T(n) \in \Omega(f(n))$. There could be a worse input that grows faster than $T(n)$, and that input would also be lower bounded by $c_3 f(n)$.

3. Writing recurrences. Here's an algorithm that you may (or may not) have seen before:

```
BinarySearch(A, x, i, j) // A is an array of length n
   if j < i return false
   mid = (i+j)/2
   if A[mid] == x return true
   if A[mid] > x return BinarySearch(A, x, i, mid-1)
   else return BinarySearch(A, x, mid+1, j)
```

Write a recurrence for $T(n)$, the runtime of `BinarySearch`. You do not need to solve your recurrence (but feel free to if you want the practice!)

Solution:

$T(n) = T(n/2) + O(1)$

Explanation: Let n be the size of the array on any given iteration of the recursive function. Then each time through the function, you will perform a comparison and (if you haven't found x) search an array of seize $n/2$.

This recurrence turns out to be $T(n) = O(\log(n))$.