Practice with Induction

1. Prove that for all $n > 0$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

2. Prove that for all $n > 0$, $\sum_{i=0}^{n} x^i = \frac{1-x^{n+1}}{1-x}$ if $|x| < 1$. This is called the finite geometric series.

Note that if we take the limit of this series as $n \to \infty$, we get $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$. This is called the infinite geometric series, and is another useful summation identity.

3. Let $T(n) = \begin{cases} T(n/2) + c_1 n & n > 1 \\ c_2 & n = 1 \end{cases}$. Prove that $T(n) \in O(n)$.

4. Let $T(n) = \begin{cases} 2T(n-1) + n & n > 1 \\ 1 & n = 1 \end{cases}$. Prove that $T(n) = 2^{n+1} - n - 2$.